Behaviour of Gradient Coils for MRI Designed with Minimised Maximum Current Density

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***Abstract*—The design of gradient coils for magnetic resonance imaging (MRI) is a well-known field synthesis inverse problem. A new method to design gradient and shim coils was recently presented that spreads out close wires using a deterministic algorithm. This has the effect of being able to increase the strength of the coil when limited by a minimum wire size. Also, it can be used to reduce the peak temperature in a coil. Here we investigate the behaviour of such coils on the interval between standard minimum power and the new minimax current density coils. Performance properties and heating experiments and simulations were performed and the results analysed.**

I. Introduction

Standard methods for field synthesis are ill-posed inverse problems that are commonly solved using Tikhonov regularisation. Minimising the stored magnetic energy or power dissipation are physically-meaningful examples of weighted Tikhonov regularisation. Previously, we reported the "minimax current density" coil design method [1] that enables direct control over the maximum current density in a coil. This method is applicable to the design of low frequency coils such as gradient and shim coils in MRI. It is useful for two principal reasons: firstly a reduction in maximum current density reduces the peak temperature reached in the coil, and secondly it permits the design of the strongest possible magnetic field to be produced when limited by the size of the coil surface and the minimum wire size. In some cases the efficiency of a minimax$|j|$ (one designed with minimum maximum current density) coil can be double that of the next best minimum power design. Experiments have demonstrated qualitatively the reduction in peak temperature of minimax$|j|$ coils [2]. In this work we explore in more detail, and quantitatively, the behaviour of minimax$|j|$ coils. Results pertain to a short, cylindrical X-gradient coil, but it is of course possible to repeat this investigation for other types of coil [1]. To assess the gradient coil performance we use Biot-Savart simulation, inductance and resistance simulation and experimental and simulated heating results. To simulate the heating we use the recently proposed method of While et al. [3].

II. Methods

A current density, $\mathbf{J}(\mathbf{r})$, is required that flows on a thin surface of arbitrary shape. Since $\nabla \cdot \mathbf{J}(\mathbf{r}) = 0$, we represent the current density by its scalar stream function, $\psi(\mathbf{r})$, over the surface such that $\mathbf{J}(\mathbf{r}) = \nabla \times [\psi(\mathbf{r})\hat{\mathbf{n}}(\mathbf{r})]$. Equispaced contours of $\psi(\mathbf{r})$ then give the wires of the coil. $\psi(\mathbf{r})$ is parameterised by basis functions, $\Psi(\mathbf{r})$, and their weights, ψ, where $\psi(\mathbf{r}) = \sum_i \psi_i \Psi_i(\mathbf{r})$. A range of gradient coils were designed by minimising the following optimisation functional:

$$U(\psi) = f(\psi) + \beta P(\psi) + \gamma \|j(\psi)\|_\infty \quad (1)$$

where f is the magnetic field error term and is the sum-of-squares of the difference between the target field and the field from the coil. P is the total amount of power dissipated by the gradient coil by Joule heating. j is the magnitude of the current density and $\|j\|_\infty$ is the maximum value of the current density magnitude, written here as the infinity-norm. β and γ are user-defined parameters used to trade-off minimisation of each term.

Whilst several parameterisations are possible [1], the basis functions, $\Psi(\mathbf{r})$, for this study have sinusoidal form in ϕ and are truncated sinusoids in z [4], [5]. 10 harmonics were used in ϕ and 20 in z, giving a total of 200 basis functions. By setting $\gamma = 0$ Eq. 1 can be solved inverting the matrix equation of $\partial U/\partial \psi = 0$ (as it is Tikhonov regularised) to produce a min(P) coil. However, by setting $\beta = 0$ we get a minimax$|j|$ coil and Eq. 1 is solved by a deterministic optimisation algorithm [1]. $\|j\|_\infty$ cannot be differentiated with respect to ψ so $\partial U/\partial \psi$ cannot be defined. A range of coils between these two extrema were designed (maintaining a maximum field error of 5%) and tested. Min(P) and minimax$|j|$ coils were constructed in such a way that the peak temperature could be measured experimentally using a thermal imaging camera (NEC F30). Temperature simulations were validated against measurements obtained from the min(P) and minimax$|j|$ coils and performed additionally for the intermediate coils to provide a quantitative prediction of their peak temperatures.

III. Results

Figure 2 a) shows a graph of the trade-off parameters, β and γ required to maintain 5% field accuracy. The appropriate figure-of-merit (FoM) with which to measure the performance of min(P) coils is η^2/R (where η is the coils efficiency and R is its resistance) since it gives a measure of the amount of magnetic field that can be generated, normalised

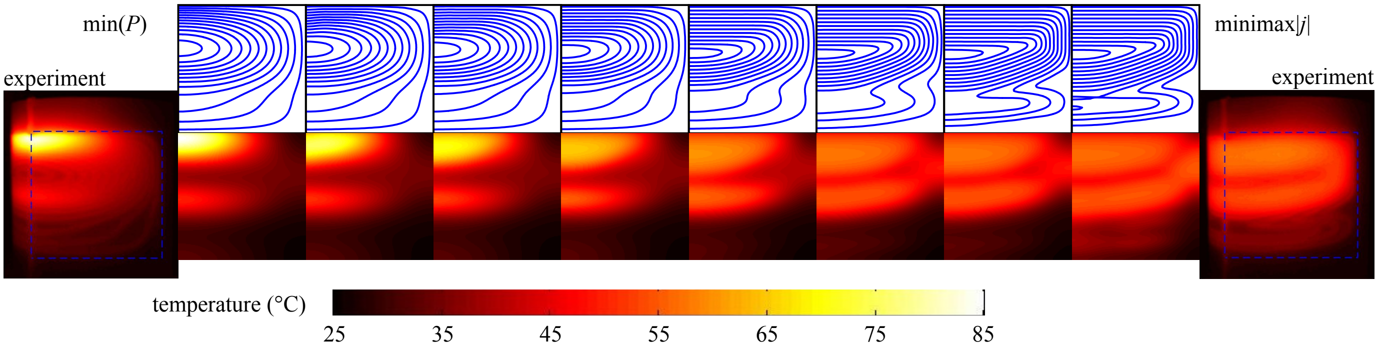

Fig. 1. Top row: wire paths for one octant of the X-gradients ranging from min(P) at the left to minimax$|j|$ at the right. Bottom row: simulated temperature maps of the corresponding coil with experimental data at the far left and right.

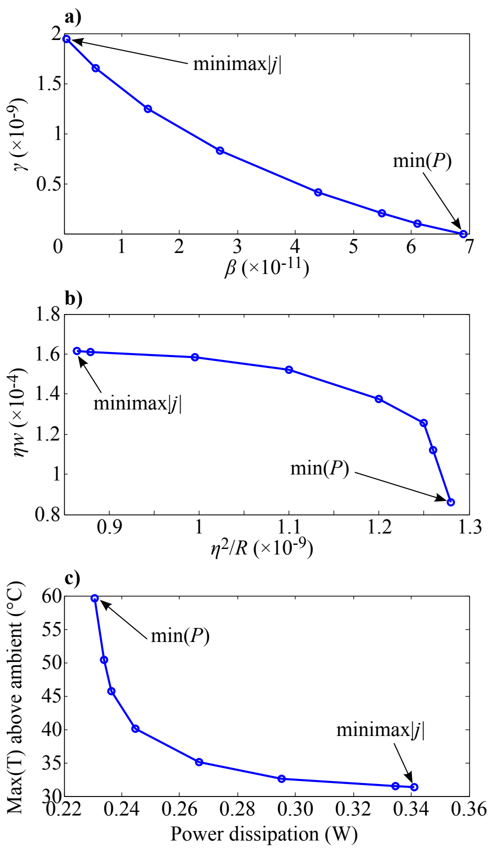

Fig. 2. a) the trade-off parameters, β and γ, that ensure max field error = 5%, b) min(P) FoM against minimax$|j|$ FoM and c) maximum temperature, max(T), as function of total power, P, dissipation of the coils.

to the amount of power dissipation. For minimax$|j|$ coils an appropriate FoM is ηw because this gives a measure of the amount of magnetic field that can be generated, normalised by the minimum spacing between wires, w. An equivalent FoM could be $\eta / \max(j)$. Figure 2 b) shows the behaviour of these two FoMs for the range of coils. Figure 1 shows the wire-paths for the coils in the range as well as their simulated temperature distribution and measured temperature for the min(P) and minimax$|j|$ coils. Figure 2 c) shows how the maximum temperature, max(T), varies with the total power.

IV. Discussion and Conclusions

Minimum inductance and power are energy terms that are usually included in the design of gradient and shim coils for MRI and are well-studied techniques [6]. They both have a natural regularising effect on the ill-posed field synthesis inverse problem that is “coil design” and are usually simply inverted because they are quadratic with respect to the solutions. $\|j\|_\infty$ is a new energy term that can be included in the design of gradient and shim coils and as such it is interesting to investigate its behaviour. Here, the behaviour of one type of coil was investigated for varying amounts of max$|j|$ minimisation. It would also be interesting to study this approach with different types of coils. The return paths (at the top of the coil) of these short cylindrical X-gradients are quite restricted and therefore the minimax$|j|$ technique has a considerable effect. The relationship between β and γ is non-linear but smooth and monotonic. Figures 2 b) and c) show that a small amount of one parameter can have a large impact on the design; i.e. a small amount of γ added to a min(P) coil significantly increases ηw (and decreases max(T)) but has little effect on η^2/R (or P). In this case the minimax$|j|$ coil can be made to be double the strength of the min(P) coil if constrained by a minimum wire spacing. Similarly, a small amount of β added to a minimax$|j|$ coil greatly reduces its power dissipation whilst maintaining low peak temperature.

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